# Simplified Methods of Predicting Aircraft Rolling Moments Due to Vortex Encounters

Timothy M. Barrows*
*U.S. Department of Transportation, Transportation Systems Center, Cambridge, Mass.*

**Computational methods suitable for fast and accurate prediction of rolling moments on aircraft encountering wake vortices are presented. Appropriate modifications to strip theory are developed which account for the effects of finite wingspan. It is shown that in the case of an elliptic wing the aspect ratio correction to the lift curve slope should be based on the semispan. A reciprocal theorem is used to relate the rolling moment on a wing in an arbitrary downwash field to that on a wing in steady rolling motion. Calculations are presented for a wing encountering a vortex with a Betz velocity distribution. It is shown that the ratio of the spans of the generating and encountering aircraft is the most significant parameter in determining the possible hazard.**

## Nomenclature

$AR$ = aspect ratio
$A_n$ = Fourier coefficient, Eq. (2)
$a$ = lift curve slope, per rad
$a_0$ = two-dimensional lift curve slope
$b$ = wingspan
$c$ = chord
$c_{av}$ = average chord
$c_0$ = root chord
$c_m$ = Fourier component, Eq. (20)
$c_\ell$ = section lift coefficient
$C_\ell$ = roll moment coefficient, $L/qSb$
$C_{\ell_p}$ = roll damping derivative, $\partial C_\ell/\partial\,(pb/2V)$
$D$ = danger coefficient, Eq. (43)
$f$ = vortex describing function, Eq. (39)
$I_n$ = integral, Eq. (25)
$I(x)$ = hazard integral, Eq. (46)
$\ell$ = section lift
$L$ = roll moment
$p$ = roll rate
$\hat{p}$ = maximum normalized roll rate, Eq. (44)
$\bar{r}$ = distance from vortex center, ft $2\bar{r}/b_g$
$r$ = $2\bar{r}/b_g$
$S$ = wing area
$V_\theta$ = tangential velocity of vortex
$V$ = wing forward velocity
$w$ = vertical velocity/$V$
$\bar{x}$ = horizontal displacement, ft, Eq. (41)
$x$ = $2\bar{x}/b_e$
$\bar{y}$ = spanwise station, ft
$y$ = $2\bar{y}/b_e$
$\alpha_{ZL}$ = section angle of attack measured from zero lift
$\gamma$ = weighting function, Eq. (19)
$\Gamma$ = spanwise circulation
$\theta$ = spanwise position variable, Eq. (1)
$\lambda$ = tip chord/root chord
$\epsilon$ = planform parameter, Eq. (22)
$\tau$ = Glauert correction factor, Eq. (15)

*Subscripts*

$e$ = encountering wing
$g$ = generating wing
$m,n$ = Fourier subscripts
$o$ = conditions at wing center
$v$ = vortex

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*Mechanical Engineer. Member AIAA.

## I. Introduction

RECENTLY, a number of investigators have formulated computer models with the purpose of analyzing the dynamics of an aircraft penetrating a wake vortex. Iverson and Bernstein[1] developed a three-degree-of-freedom analogue simulation which used simple strip theory to predict rolling moments. Harlan and Madden[2] produced a hybrid computer program with a digital computation of the lift and rolling moments. A modified strip integration is used, rather than lifting surface theory, in order to allow the digital program to keep up with the real-time analogue computation of aircraft motions. To account for effects of finite span, the sectional lift curve slope at each station along the wing is weighted in such a way as to duplicate various known stability derivatives. Johnson, Teper, and Redeiss[3] utilized a similar technique. Nelson[4] performed some investigations using lifting surface theory and concluded that strip theory is more appropriate for the vortex encounter problem. His approach utilized a different aspect ratio correction for symmetric and antisymmetric components of the wing lift, which as shown herein is essential for accurate results. Finally, Jenkins and Hackett[5] use vortex lattice theory to predict aircraft forces and moments for use with a flight simulator. Their approach included some important effects of induced lift on the yawing moments. However, they too encountered some drawbacks due to the need for real time computation; namely, the effects of local stall had to be ignored.

These efforts all reveal a need for fast and reasonably accurate methods of calculating forces and moments due to vortex encounters. In this report a technique using a simple spanwise integration is presented. The relation to strip theory is shown, and the degree of accuracy of the method is clearly indicated.

## II. Lifting Line Analysis

Consider an unswept elliptical wing encountering a velocity field in which the downwash varies as a function of span, as shown in Fig. 1. This problem can be solved using the classical approach of Prandtl and Glauert.[6]

We employ the following coordinate transformation

$$\bar{y} = (b/2)\cos\theta \tag{1}$$

The distribution of circulation on the wing may be written as a Glauert series

$$\Gamma = Vb\sum A_n \sin n\theta \tag{2}$$

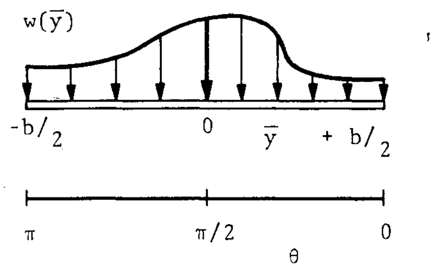


Fig. 1 **Wing encountering arbitrary downwash field.**

which guarantees that $\Gamma=0$ at the wingtips as required. The rolling moment on the wing is given by

$$C_\ell=\frac{2}{VSb}\int_{-b/2}^{+b/2}\Gamma\bar{y}\mathrm{d}\bar{y}=\frac{b^2}{2S}\int_0^\pi\sum A_n\sin n\theta\cos\theta\sin\theta\mathrm{d}\theta$$

$$=\frac{AR}{4}\int_0^\pi\sum A_n\sin n\theta\sin 2\theta\mathrm{d}\theta$$

The integral in this expression is zero for all values of $n$ except $n=2$, which gives

$$C_\ell=\pi AR\,A_2/8 \tag{3}$$

Therefore, in order to predict rolling moments it is not necessary to know all of the coefficients $A_n$ in the series (2); only the second term is required. This is analogous to the more familiar result that all of the lift on the wing may be found from the first term, $A_1$.

Lifting line theory[6] gives us the following expression for the circulation

$$\Gamma=\pi cV\left[w(y)+\alpha_{ZL}(y)-\frac{1}{4\pi V}\int\frac{\mathrm{d}\Gamma}{\mathrm{d}y_1}\times\frac{\mathrm{d}y_1}{(y-y_1)}\right] \tag{4}$$

$\alpha_{ZL}$ is the angle of attack of each wing section, measured from the incidence of zero lift. This term results in a symmetrical distribution of lift and can be neglected for purposes of determining rolling moment. We insert Eqs. (1) and (2) into Eq. (4), and make use of the following

$$\int_0^\pi\frac{\cos n\theta_1\mathrm{d}\theta_1}{\cos\theta-\cos\theta_1}=\frac{\pi\sin n\theta}{\sin\theta}\qquad c=c_1\sin\theta \tag{5}$$

This results in the classical algebraic form of the lifting line equation

$$\sum A_n\sin n\theta\left[1+\frac{n\pi c}{2b\sin\theta}\right]=\frac{\pi\,cw(\theta)}{b} \tag{6}$$

We are only interested in solving the above for the coefficient $A_2$. Multiplying both sides by $\sin 2\theta$ and integrating over the interval from 0 to $\pi$, we obtain

$$A_2\frac{\pi}{2}\left(1+\frac{\pi c_1}{b}\right)=\frac{\pi c_1}{b}\int_0^\pi w\sin\theta\sin 2\theta\mathrm{d}\theta \tag{7}$$

Let us define

$$I_2\equiv\int_0^\pi\sin\theta\sin 2\theta\, w(\theta)\,\mathrm{d}\theta \tag{8}$$

For an elliptic wing, we have

$$\pi c_1/b=4/AR \tag{9}$$

Using Eq. (3), we obtain

$$C_\ell=\frac{I_2}{(1+4/AR)} \tag{10}$$

It is very informative to compare this to the result from strip theory, which computes the rolling moment coefficient by means of a simple spanwise integration

$$C_{\ell_{\text{strip}}}=\frac{1}{Sb}\int_{-b/2}^{+b/2}a_0 cw\bar{y}\mathrm{d}\bar{y}=\frac{c_1 b\,a_0}{8S}I_2 \tag{11}$$

Using the theoretical value for $a_0$ of $2\pi$, we obtain

$$C_{\ell_{\text{strip}}}=I_2 \tag{12}$$

The relation between this result and the more accurate expression [Eq. (10)] is very simply

$$C_\ell=\frac{C_{\ell_{\text{strip}}}}{(1+4/AR)} \tag{13}$$

This result may be interpreted as an aspect ratio correction to the two-dimensional lift curve slope $a_0$. That is, if we choose

$$a=\frac{a_0}{(1+4/AR)} \tag{14}$$

then a strip theory integration of the spanwise lift will yield a correct value of the rolling moment. This result is applicable, within the limits of lifting line theory, to an unswept elliptic wing with any distribution of camber and twist encountering any downwash field, provided that the entire wingspan remains free of stall.

So far as could be determined, this simple result has not been previously derived in the literature. It is curious that, apparently by conjecture, Fung[7] presented the following aspect ratio correction for unswept wings of a general planform

$$a=\frac{a_0}{1+(a_0/\pi AR_e)(1+\tau)} \tag{15}$$

where

$AR_e$ $=AR$ (for symmetrical lift distribution)
$AR_e$ $=AR/2$ (for antisymmetrical lift distribution)
$\tau$ = correction factor computed by Glauert
$\tau$ = 0 for elliptic lift distribution.

For the symmetrical case this is the classical result from Prandtl and Glauert.[6] In the antisymmetric case, using the theoretical value for $a_0$ of $2\pi$, and setting $\tau=0$, we see that Eq. (15) is identical to Eq. (14). Fung's argument was that if the lift is distributed antisymmetrically, with zero lift at midspan, then each of the two halves of the distribution appear similar to an elliptic distribution with a span equal to half of the actual wingspan. The argument itself is not entirely correct, since in fact the distribution of each side can vary considerably from elliptic, depending on the nature of the oncoming downwash field, but the end product is a result which is correct to first order for any distribution of lift.

Unfortunately there is no theoretical basis for Fung's conjecture that $\tau$, computed on the basis of an untwisted wing at constant angle of attack, can be used to predict the effect of a nonelliptic planform on the antisymmetric lift. The essence of the problem is that applying Eq. (13) to a wing planform with finite chordlength at the tip produces a lift distribution which is likewise finite at the tip, which is in violation of the condition that $\Gamma(\pm b/2)=0$. Thus for the tapered wings of typical aircraft there is an error of unknown magnitude associated with this violation. We find, for example, that Eq. (13) tends to overestimate the roll damping of rectangular wings by a considerable amount.

A straightforward approach to the problem would consist of computing the correction $\tau$ association with nonelliptic planforms for antisymmetric lift. We immediately find that $\tau$ varies according to the oncoming downwash field, just as it does in the symmetric case.

## III. The Reciprocal Theorem

Fortunately, we can circumvent this problem by utilizing a reciprocal theorem developed by Heaslet and Spreiter,[8] which relates the rolling moment on a wing in an arbitrary downwash field to that on a fictitious wing in steady rolling motion. This theorem may be stated as follows: "The rolling moment on a wing encountering an arbitrary downwash field is equal to the integral over the span of the product of the local angle of attack and the sectional lift at the corresponding spanwise station of a flat-plate wing of identical planform, which is rolling at a rate $p=2V/b$."

Essentially, this theorem states that a modified strip integration may be used to compute the rolling moment, which becomes

$$L=\int_{-b/2}^{+b/2}[\ell(y)]_{w=y}w(\bar{y})\mathrm{d}\bar{y} \tag{16}$$

Eggleston and Diederich[9] have cast the corresponding rolling moment coefficient in the form

$$C_\ell=\frac{-C_{\ell_p}}{4}\int_{-1}^{+1}\gamma(y)w(y)\mathrm{d}y \tag{17}$$

where

$$\gamma(y)\equiv-[C_\ell(y)c(y)/C_{\ell_p}c_{av}]_{w=y} \tag{18}$$

By virtue of this definition

$$\int_0^1\gamma(y)y\mathrm{d}y=2 \tag{19}$$

The theorem in this form has been used to analyze the problem of aircraft penetrating random turbulence. While it is possible to obtain the weighting function $\gamma$ from lifting surface theory, Weissinger theory, or any of the numerous techniques available for wing analysis, the indeterminate nature of random turbulence negates the usefulness of highly refined calculations of $\gamma$. Usually $\gamma$ is calculated from simple theories, often by assuming an elliptic or parabolic distribution when the wing is at a uniform angle of attack, as has been done by Eggleston and Diederich[9] and by Franklin.[10] One exception is Jackson and Wherry,[11] who went to the trouble of using Weissinger theory to compute $\gamma$ for gust analysis of the B-47.

The wake vortex problem is comparatively well defined, so that the greater accuracy available from more sophisticated theories may be useful. Fairly accurate predictions of the rolling moment can be made if the characteristics of the vortex generating aircraft are known. The major indeterminant in the problem then becomes the ensuing control action, which may be expected to vary from pilot to pilot and according to the exact circumstances of the encounter. Thus, the nature of the problem warrants an analysis of roll moments which is more refined than simple strip theory but not as complicated as a full-blown lifting surface theory.

The wing planform may be described as a harmonic series

$$c=\sum c_m\sin m\theta \tag{20}$$

In the interest of maintaining a simple solution for the rolling moment which can be expressed in closed form, we consider one degree of complication beyond the elliptic wing, in which only two terms of this series are present, and one of these is very small

$$c=c_1(\sin\theta+\epsilon\sin 3\theta) \tag{21}$$

$$\epsilon\equiv c_3/c_1 \tag{22}$$

Equation (6) becomes

$$\sum A_n\sin n\theta\left[1+\frac{n\pi}{2b\sin\theta}\sum c_m\sin m\theta\right]=\frac{\pi cw}{b} \tag{23}$$

If we multiply this by sin $2\theta$ and integrate from 0 to $\pi$ we obtain an equation analogous to Eq. (7). We can also multiply by sin $4\theta$, integrate, and obtain another equation. These two equations have the form

$$a_{11}A_2+\epsilon a_{12}A_4=I_2 \tag{24a}$$

$$\epsilon a_{21}A_2+a_{22}A_4=I_4 \tag{24b}$$

$$I_n\equiv\int_0^\pi\left(\frac{c}{c_1}\right)w\sin n\theta\mathrm{d}\theta \tag{25}$$

$$a_{11}=(\pi AR/8)[1+(4/AR)(1+\epsilon)] \tag{26}$$

$$a_{12}=\pi \tag{27}$$

$$a_{21}=\pi/2 \tag{28}$$

$$a_{22}=(\pi AR/8)[1+(8/AR)(1+\epsilon)] \tag{29}$$

For steady roll at a rate $p=2V/b$ we have

$$w=2\bar{y}/b=\cos\theta \tag{30}$$

This yields

$$I_4=\pi\epsilon/4$$

If we neglect terms of order $\epsilon^2$, the solution to the set of Eqs. (24) for $A_2$ becomes very simple

$$A_2=I_2/a_{11}$$

Using Eqs. (3), (12), and (26), the rolling moment coefficient becomes

$$C_\ell=\frac{C_{\ell_{\mathrm{strip}}}}{1+(4/AR)(1+\epsilon)} \tag{31}$$

We have not yet dealt decisively with the question of whether we can perform a strip integration on a wing with a finite chord at the tip, as opposed to the rounded tips obtained from the series (21). The strip integration (11) with $w=\cos\theta$ may be written as follows

$$C_{\ell_{\mathrm{strip}}}\sim\int_0^\pi c\cos^2\theta\sin\theta\mathrm{d}\theta\sim\int_0^\pi c\cos\theta\sin 2\theta\mathrm{d}\theta \tag{32}$$

Using the product rule, we have

$$2\sin 2\theta\cos\theta=\sin\theta+\sin 3\theta$$

Thus

$$C_{\ell_{\mathrm{strip}}}\sim\int_0^\pi\sum c_m\sin m\theta(\sin\theta+\sin 3\theta)\,\mathrm{d}\theta \tag{33}$$

In this form it is clear that only the terms $c_1$ and $c_3$ make a finite contribution. Including the infinite number of additional Fourier components necessary to produce a squared wingtip will not produce any net effect on the integral, which means that we can perform our strip integration using either the actual planform or a two-term Fourier description and obtain identical results. For any distribution of downwash other than solid body rotation ($w=y$) this statement no longer applies, in which case the reciprocal relation may be utilized.

The roll coefficient for the case of $w=y$ may be interpreted as the roll damping derivative $C_{\ell_p}$, which has been calculated by Pearson and Jones[12] for tapered wings. It is a rather straightforward matter to calculate the Fourier coefficients of such wings as a function of $\lambda$, the ratio of the root chord to the tip chord

$$c_1 = (2/\pi)(1+\lambda)c_0 \tag{34}$$

$$c_3 = (2/\pi)[(3\lambda-1)/3]c_0 \tag{35}$$

$$\epsilon = (3\lambda-1)/3(1+\lambda) \tag{36}$$

The parameter $\epsilon$ has a value of zero when the taper ratio is 1/3. Most commercial subsonic jets have approximately this amount of taper, so that in practical cases of interest $\epsilon$ is indeed very small. (For low-speed general aviation aircraft $\lambda$ is typically 0.5, giving $\epsilon = 1/9$.) Performing the integration indicated in Eq. (31), we obtain

$$C_{\ell_p} = \frac{a_0}{12}\,\frac{1+3\lambda}{1+\lambda}\,\frac{AR}{[AR+4(1+\epsilon)]} \tag{37}$$

Figure 2 shows a comparison of Eq. (37) with the values computed by Pearson and Jones for various values of $\lambda$ and $AR$, using a value for $a_0$ of 5.67.

The weighting function $\gamma$ may be computed from the definition Eq. (18) and the series (2)

$$\gamma = \frac{2b\sum A_n \sin n\theta}{-C_{\ell_p} c_{av}}$$

After some algebra this becomes

$$\gamma = \frac{16}{\pi}\left[\sin 2\theta + \frac{\epsilon AR \sin 4\theta}{(1+\epsilon)[AR+8(1+\epsilon)]}\right] \tag{38}$$

For purposes of calculating vortex encounter motions, the second term in Eq. (38) is rarely important. Neglecting this term has no effect on the net rolling moment if the oncoming downwash field is in solid body rotation, but produces some error for any other distribution. A specific example can be chosen to illustrate when this term becomes important. For a triangular wing planform with $AR=6$ in a point vortex ($w=1/y$), including the second term increases the calculated rolling moment by 20%.

Eggleston and Deiderich[9] present various formulas for $\gamma$ based on assumed loading under constant angle of attack. We see here that their formula for elliptic loading, identical to the first term of Eq. (38), is by far the most applicable to typical planforms of interest. The second term in Eq. (38) can be used as a means of determining the degree of accuracy of the first term alone.

The manner in which $\gamma$ is defined automatically contains a first-order correction for the effects of sweep, changes in the planform due to flaps, tip tanks, etc. (assuming the availability of a value for $C_{\ell_p}$ which includes all these effects). If it is determined that an accurate value for the second term of Eqs. (38) is desired, some modification to account for these factors may be necessary. Wing sweep tends to distribute the

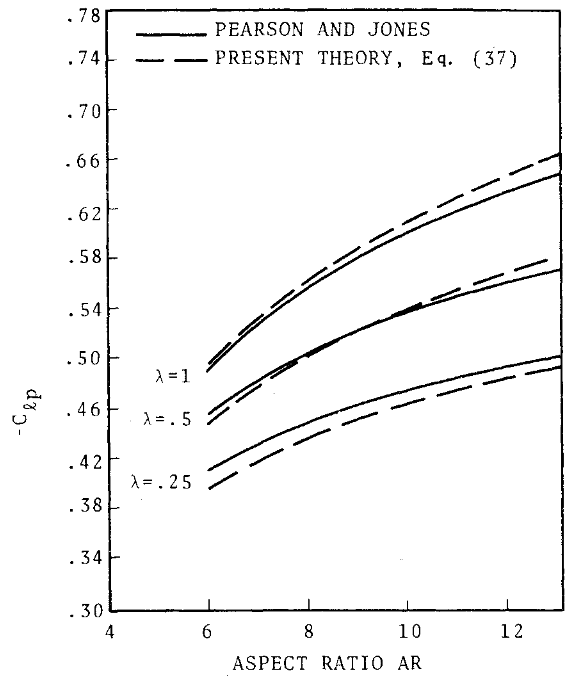


**Fig. 2 Roll damping derivitive as a function of aspect ratio.**

lift toward the outboard sections, thus making the second term more positive. The magnitude of this effect can be estimated from the work of Bird.[13] Flaps only have an effect if they change the lift curve slope, through an increase in the effective chord. This may be accounted for by computing the coefficients $c_1$ and $c_3$ based on the effective planform of the wing with the flaps extended.

## IV. Wake Vortex Encounter

Up to this point very little has been said about the actual distribution of downwash which is likely to be encountered. Perhaps the best model of a vortex structure is the one proposed by Betz[4] and resurrected by Donaldson.[15] An approximation to the tangential velocity associated with this model for an elliptically loaded wing has the following form

$$V_\theta(\bar{r}) = (\Gamma_0/2\pi\bar{r})f(r) \tag{39}$$

where

$$f(r) = (3r - 9r^2/4)^{1/2} \quad \text{for } r<2/3$$

$$f(r) = 1 \quad \text{for } r>2/3$$

$$r = \frac{2\bar{r}}{b_g}$$

and $b_g$ = span of generating aircraft.

Assuming an elliptic lift distribution on the generating aircraft, the circulation $\Gamma_0$ is related to the parameters of the aircraft as follows

$$\Gamma_0 = \frac{2}{\pi}\left(\frac{VC_LS}{b}\right)_g \tag{40}$$

The rolling moment coefficient induced by the vortex as predicted by Eq. (17) becomes

$$C_{\ell_v} = \frac{C_{\ell_p}}{2\pi^2}\left(\frac{C_L}{AR}\right)_g \frac{V_g}{V_e}\int_{-1}^{+1}\frac{\gamma(y)f(r)\,dy}{r}$$

Let us now assume the vortex lies in the same horizontal plane as the encountering wing, and is displaced by a

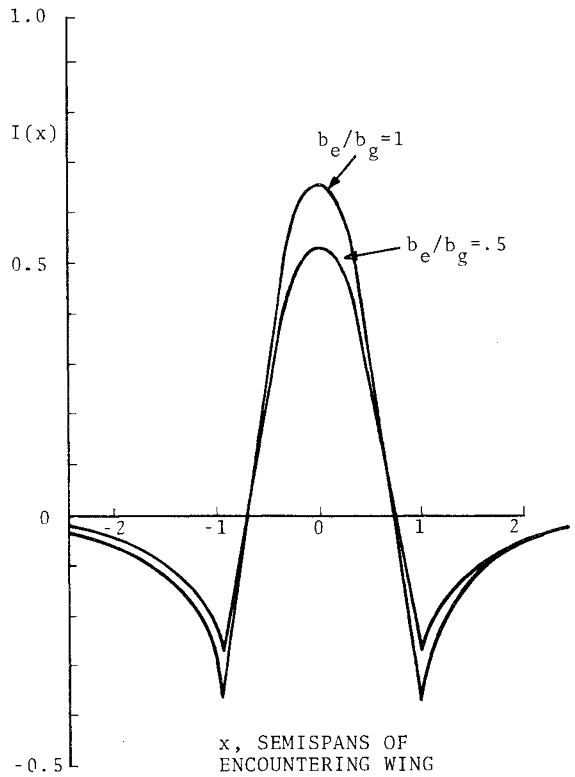


Fig. 3 Normalized rolling moment versus lateral separation, Eq. (46).

horizontal distance $\bar{x}$

$$\bar{r}=\bar{y}-\bar{x} \tag{41}$$

We nondimensionalize $\bar{x}$ by the semispan of the encountering wing

$$x=2\bar{x}/b_e \tag{42}$$

A common method of presenting the hazard associated with an encounter is to compute $D$, the ratio of the vortex induced rolling moment to the maximum control power available from the ailerons

$$D=C_{\ell_v}/C_{\ell_{max}} \tag{43}$$

When this is done we find ourselves confronted with the following quantity

$$C_{\ell_{max}}/C_{\ell_p}=(pb/2V)_{max}\equiv\hat{p} \tag{44}$$

This parameter is equal to the helix angle described by the wingtips corresponding to full aileron deflection. It is readily obtainable from flight test data and actually represents a more convenient parameter than either $C_{\ell_p}$ or $C_{\ell_{max}}$, since it generally shows only slight variation among different aircraft. Perkins and Hage[16] give the following for different types of aircraft: $\hat{p}=.07$ cargo and bomber types; $\hat{p}=.09$ fighter types.

Our final formulation for $D$ becomes

$$D=\frac{1}{\hat{p}_e}\left(\frac{C_L}{AR}\right)_g\frac{V_g b_g}{V_e b_e}I(x) \tag{45}$$

where

$$I(x)\equiv\frac{1}{2\pi^2}\int_{-1}^{+1}\frac{\gamma(y)f(r)\,\mathrm{d}y}{y-x} \tag{46}$$

and

$$r=(b_e/b_g)(y-x) \tag{47}$$

The danger factor $D$ in this form is composed of a series of three easily recognizable nondimensional factors times the integral $I(x)$ which is given in Fig. 3. The weighting function $\gamma$ is calculated assuming $\lambda=1/3$. The maximum value of $I$ is approximately 0.6 (depending on span ratio), which occurs at $x=0$, corresponding to a vortex located at the center of the wing. To develop a greater feel for the maximum value of $D$, we might assume the following typical values: $\hat{p}=0.08$; $C_L=1$; $AR=7$; $V_g=V_e$. This gives

$$D_{max}\cong b_g/b_e$$

Thus for these typical parameters a following aircraft may become uncontrollable if its span is less than that of the lead aircraft. There are several implied assumptions associated with this statement: 1) no decay of the vortex strength has taken place; 2) the vortex has no core; 3) there is no effect from the vortex shed off the opposite wingtip of the generating aircraft; and 4) the lift distribution of the generating wing is approximately elliptic and the shed vorticity rolls into a tightly wrapped vortex as prescribed by Betz.

The decay of vortex strength with time is a complex subject which is beyond the scope of this paper. The related subject of core size is likewise complicated, although the Betz model does not require the existence of a core for conservation of energy, as does a point vortex model. Only a small modification to the model is required in order to avoid infinite velocities at the center, and this produces a negligible change in the overall rolling moment.

It is a straightforward matter to compute the effect of two vortices rather than one, essentially this is a matter of superposition. However, there is some reason for interest in the single vortex encounter. First, wind shear sometimes has the effect of destroying one of the two trailing vortices, leaving an isolated vortex which has a tendency to persist for longer times. Second, the greatest potential hazard exists when the aircraft is about to land. In ground effect the two vortices tend to separate under the influence of their image vortices, and a crosswind is required to hold one or the other vortex stationary over the runway. Thus, in two situations of great interest (long vortex life and greatest potential hazard) the following aircraft could encounter a single vortex.

Finally, we come to the last assumption, that the vortex is tightly wrapped. Calculations by Donaldson[15] show that this is indeed the case when the aircraft is cruising at altitude, but during approach to landing, with gear and flaps deployed, vorticity tends to shed unevenly, often forming two or more vortices from each wing which enter a spiral pattern. The net effect is that the vortex has a less concentrated character and may produce rolling moments somewhat less than indicated by Eq. (45).

## V. Conclusions

For purposes of calculating global forces and moments during wake vortex encounters it is not necessary to use time-consuming lifting surface theories. The reciprocal theorem of Heaslet and Spreiter[8] gives a recipe for doing this with a simple spanwise integration. Ordinary strip theory can also be used for planforms of typical subsonic jets if the aspect ratio correction to the lift curve slope for antisymmetric loadings is based on the semispan rather than the span.

Figure 3 may be interpreted as the normalized rolling moment versus time seen by an aircraft as it crosses a vortex at a shallow angle. It can be seen that the largest rolling moments occur when the vortex lies within the tips of the span. With the vortex only one semispan beyond the wingtip, rolling moments are less than 10% of their maximum value. For air traffic control purposes, lateral separation criteria will undoubtedly be dictated by the uncertainties of locating the vortex rather than by the rather narrow separations which might be safe if some precise distance from the vortex could be guaranteed.

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